Tuning equation ford dynamic matrix control in siso loops

Edinzo J. Iglesias*, Marco E. Sanjuán** y Carlos A. Smith***

Resumen

El Control por Matriz Dinámica (DMC) es una de las estrategias de control avanzado que más aplicaciones industriales tiene en la actualidad. Sin embargo, la literatura presenta pocas opciones para el cálculo del parámetro de sintonización que gobierna la agresividad del controlador. Esta investigación propone una nueva ecuación de sintonización para calcular este parámetro de sintonización. Se presentan los análisis estadísticos realizados para formular la ecuación de sintonización. Para probar la eficacia de la ecuación propuesta, se presenta pruebas de rendimiento del controlador usando diferentes métodos de sintonización. Estas pruebas incluyen tanto sistemas lineales como no lineales.

Palabras claves: Control automático procesos, control por matriz dinámica, ecuación sintonización.

Abstrac

Dynamic Matrix Control (DMC) is one of the most used advanced control strategies used in industrial environments. However, the available literature does not present many alternatives to calculate the controller tuning parameter (also called suppression

Fecha de recepción: 14 de febrero de 2006
Fecha de aceptación: 26 de abril de 2006

* Ph.D. Associate professor in the Chemical Engineering Department at the University of Los Andes (Venezuela). He received his BS (1993) and his MSc (1998) in Chemical Engineering at ULA, and his doctoral degree (2006) in Chemical Engineering at the University of South Florida (USF). *iedinzo@ula.ve*

** Ph.D. Assistant professor at Universidad del Norte (Colombia), Director of the Robotics and Automation Laboratory. BS in Mechanical Engineering (1995) from Universidad del Norte, and Master of Science (1999) and Doctor of Philosophy (2002) in Engineering Science from University of South Florida.
Dirección postal: Universidad del Norte, Km 5, vía a Puerto Colombia, A.A. 1569, Barranquilla (Colombia). *msanjuan@uninorte.edu.co*

*** Ph.D. B.Sc. in Chemical Engineering from University of Florida; M.Sc. and Ph.D. in Chemical Engineering from Louisiana State University. He is the author of *Continuous Automated Process Control* (2002), and co-author of *Principles and Practice of Automated Process Control* (1984, 1997). Full Professor in the College of Engineering at University of South Florida. *csmith@eng.usf.edu*

INGENIERÍA & DESARROLLO
Número 19
Enero-Junio, 2006
ISSN: 0122-3461

factor). This research proposes a new tuning equation to calculate this parameter. The statistical analysis and regression used to develop the equation, as well as the tests used to validate it are shown. Linear and nonlinear systems were used to compare different tuning methods.
Key words: Process control, dynamic matrix control, tuning equation.

1. INTRODUCTION

Dynamic Matrix Control (DMC), originally developed by Cutler and Ramaker in 1979 [1], is a successful and widely used technique in industrial applications [2-11]. It is considered a Model Based Controller (MBC) because the prediction capability is based on the process model incorporated inside the algorithm. DMC's main characteristics are [12]:

- Uses linear step response model to predict process behavior.
- A quadratic objective performance over a finite prediction horizon is employed.
- Future plant outputs are specified to follow the set point as closed as possible.
- Optimal outputs, to track set point, are calculated using least square method.

DMC, and other MBC schemes, allows intrinsic dead time compensation because of the process model used to predict future behavior.

The matrix operations on which the DMC calculations are based can easily be extended to any number manipulated and controlled variables. For any pair of manipulated-controlled variable, a unit step response vector is required. Each of these vectors is used to form the dynamic matrix. The individual dynamic matrices are sub matrices of the global matrix. Sanjuan showed a detailed explanation about the implementation of the Dynamic Matrix Controller (DMCr) using matrix notation [13][14].

The controller output, vector ΔM can be calculated as:

$$\Delta M = (A^T A)^{-1} A^T E \qquad (1)$$

where **A** is the dynamic matrix containing the process dynamic information. **E** is the error between the set point and the actual value of the controlled variable.

Usually a suppression factor is used as a tuning parameter to adapt the aggressiveness of the controller; in that case the control move is expressed as:

$$\Delta M = (A^T A + \lambda^2 I)^{-1} A^T E \tag{2}$$

where λ is the suppression factor and **I** is the identity matrix.

The DMC control law can be also expressed in terms of the constant matrix **KIPS**:

$$\Delta M = ((A^T A + \lambda^2 I)^{-1} A^T)E = (KIPS)E \tag{3}$$

KIPS is an invariant element of the control law. It is calculated off-line and stored to be used when the DMC algorithm requires to calculate ΔM.

Although Shridhar and Cooper [15-17] have proposed equations to determine adequate values of the suppression factor, it is common industrial practice to use a trial and error procedure to choose the λ value [14]. This practice is time consuming and demands a considerable effort by control engineers because for every suppression factor the KIPS matrix must calculated and stored; with the corresponding computing time and effort involved.

Shridhar and Cooper tuning equations for SISO control loops are:

$$f = \left\{ \begin{array}{lll} \text{if} & M = 1 & 0 \\ \text{if} & M > 1 & \dfrac{M}{500}\left(\dfrac{3.5\tau}{Ts} + -\dfrac{M-1}{2}\right) \end{array} \right\} \tag{4}$$

$$\lambda_{SC} = f K_p^2 \tag{5}$$

where M is the control horizon, an integer number usually from 1 to 6. Ts is the sampling time, and is the largest value that satisfies $Ts \leq \tau/10$ and $Ts \leq to/2$. K_p is the process gain.

Recent research [18] has shown that Equations (4) and (5) predict λ values that commonly generate aggressive behavior, which could be detrimental for chemical processes with highly nonlinear behavior. Therefore, it is necessary to develop a more reliable equation to determine λ values. That is precisely the goal of this paper, to propose a new tuning equation for DMCr in SISO control loops.

The following sections present the method used to develop the new tuning equation, and the tests used to evaluate and compare the performance of standard DMCr working with different tuning methods.

2. DMCr TUNING EQUATION DESIGN

To develop the new tuning equation a factorial experiment was designed, and an analysis of variance (ANOVA) was performed to determine the variables that have a significant influence on the optimal suppression factor λ. The experiment consisted in modeling a general process as a first-order-plus dead time (FOPDT) and determine, using constrained optimization, the best λ value to minimize a cost function. The FOPDT model contains three parameters, process gain, K_p, time constant, τ, and dead time *to*.

A total of 3^5 simulations were performed for the experiment, corresponding to the 243 possible combinations of factor values choose for the study. No replicates were necessary because this experiment is a deterministic computational test where repetitions of factor levels provide the same result every time. Table 1 shows the three levels used in the factorial experiment for each factor.

Table 1

Factors used to perform the designed experiments

Level	K_p	λ	to/λ	Ts/τ	Γ
Low	0.5	1	0.2	0.05	2
Medium	1.5	3	0.6	0.1	5
High	2.5	5	1	0.15	8

Γ is a weighted parameter used in the cost function. The cost function used was defined using a combination of the Integral of the Absolute value of the Error (IAE) and the Integral of the Manipulated Valve signal (IMV). This cost function or Performance Parameter (PP) is expressed as:

$$PP = \int_0^\infty \left| e(t) \right| \, dt + \Gamma \int_0^\infty \left| m(t) \right| \, dt \qquad (6)$$

The optimal suppression factor for each experiment condition is defined as the λ value which minimizes Equation (6). This cost function was selected after some attempts using only IAE as performance parameter; for many experiment conditions the minimum IAE resulted in a non desirable oscillatory behavior. Adding the IMV, the optimum suppression factors minimize the oscillatory behavior.

To run the experiment and determine the optimal suppression factor for each condition, a Matlab program was developed using the Optimization and Statistical toolboxes available in Matlab Release 6.5. For every experiment condition a set point change equal to +10%TO at time 10 s was introduced. Later, at time 40 s a disturbance of +10%TO was introduced into the process. These two changes were made with the purpose of finding optimal values of λ useful for set point changes and disturbances affecting the process; therefore, the tuning equation developed can be used in both cases. Figure 1 shows a typical test performed to find the optimal value of suppression factor to minimize Equation (6).

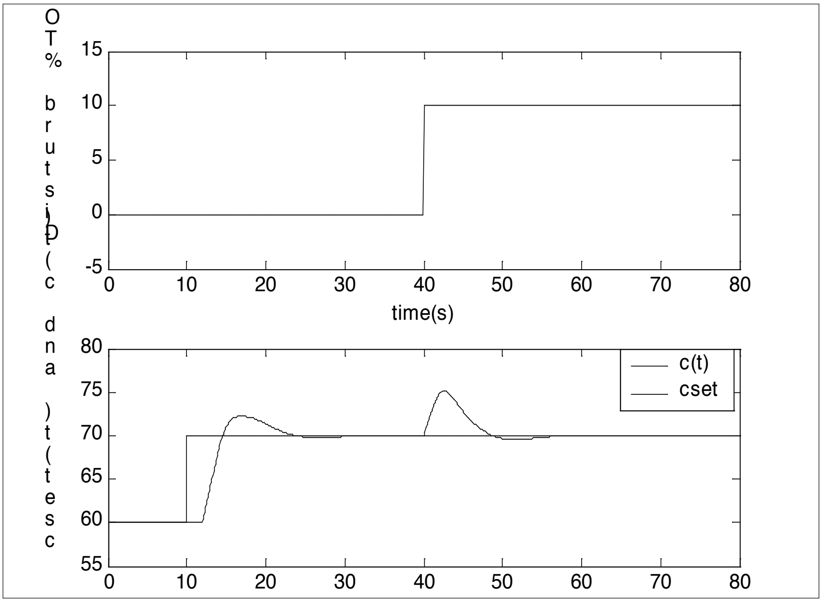

Figure 1

Example of set point change and disturbance used with FOPDT to find optimal suppression factor for DMCr

For all experiment conditions, the DMCr was implemented using a Control Horizon (CH) equal to 5, a sampling time equivalent to 0.1T and a Sampling Size (SS) to build matrix **A,** equivalent to $4\tau + to$; as literature recommends (see references [13] and [14]). This sampling size is large enough to capture process dynamic characteristics, but no so large to obtain a sluggish DMC response.

Once the complete set of optimal values of the suppression factor was found, an analysis of variance (ANOVA) was performed. The ANOVA allows determining the most significant factors for the optimal tuning. Only main effects and second order interaction were considered. Table 2 shows the ANOVA table for the experiment.

Table 2
ANOVA table for optimal suppression factor

Source	Sum Sq.	DoF	Mean Sq.	F	P value
K_P	151.5478	2	75.7739	447.9556	0
τ	4.417000	2	2.2084	13.06000	0
to/τ	14.78930	2	7.3946	43.71510	2.2204e-016
Ts/τ	7.64030	2	3.8202	22.58380	1.5546e-009
Γ	1.68550	2	0.84273	4.982000	0.0077736
K_p*τ	2.28200	4	0.5705	3.372700	0.0107730
K_p*to/τ	2.52410	4	0.63102	3.730400	0.0060036
K_p*Ts/τ	1.18480	4	0.29619	1.751000	0.1404500
K_p*Γ	0.12735	4	0.031837	0.188210	0.9443400
τ*to/τ	2.92150	4	0.73037	4.317700	0.0022875
τ*Ts/τ	1.17940	4	0.29485	1.743100	0.1421300
τ*Γ	0.36383	4	0.090959	0.537720	0.7081900
to/τ *Ts/τ	3.98440	4	0.9961	5.888700	0.0001720
to/τ *Γ	0.47614	4	0.11904	0.703710	0.5903000
Ts/τ *Γ	0.26025	4	0.065063	0.384630	0.8194700
Error	32.4778	192	0.16915		
Total	227.8611	242			
where					
DoF: Degree of Freedom					
F: Test Statistic F					

The significant factors are those with a P value less than 0.05. Therefore, the significant factors are: K_p, τ, to/τ, Ts/τ, Γ, K_p/τ, K_p*to/τ, *to* and to/τ*Ts/τ. Using this information and the set of optimal suppression factor available, nonlinear regressions were performed, using many possible combinations of the significant factors, until a good correlation coefficient was obtained. The tuning equation that best fit the optimal values of the suppression factor ($R^2 = 0.9595$) is:

$$\lambda = 1.631 K_p \left(\frac{to}{\tau} \right)^{0.4094} \qquad (7)$$

Equation 7 must be applied using a ratio sampling time to process time constant (Ts/τ) equal to 0.1.

3. SIMULATION RESULTS

To validate the new tuning equation, suppression factors were calculated using the Shridhar and Cooper tuning equations, Equation (7), and the optimal value determine by Matlab optimization toolbox. The DMCr performance was compared using those values.

For the following FOPDT process:

$$\frac{C(s)}{M(s)} = \frac{0.5e^{-0.2s}}{s+1} \tag{8}$$

The suppression factor using Shridhar and Cooper [15] λ_{SC} is:

$$\lambda_{SC} = 0.0875 \tag{9}$$

Equation (7) gives:

$$\lambda = 0.4220 \tag{10}$$

Using optimization methods, the value for λ is:

$$\lambda_{OPT} = 0.325 \tag{11}$$

Figure 2 shows the performance comparison. The figure shows that the controller response using the Shridhar and Cooper value generates oscillatory behavior. The suppression factor predicted from these equations is the smaller one, which implies the most aggressive controller behavior. This result seems to be a general tendency of the Shridhar and Cooper equations because they always generated the smaller λ. Figure 2 shows that the response obtained using the suppression factor calculated with Equation (7) is almost the same when the optimal value of λ is used.

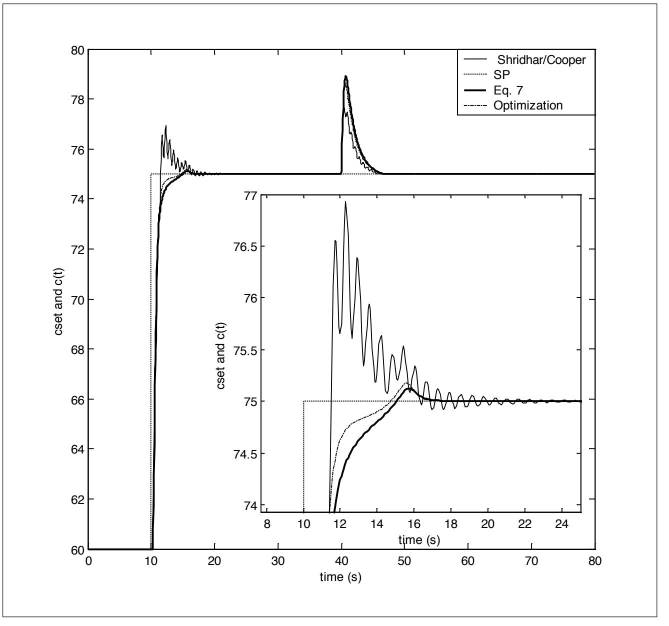

Figure 2

Performance comparison using different tuning methods

As a second test, a simulation of the mixing process shown in Figure 3 was used.

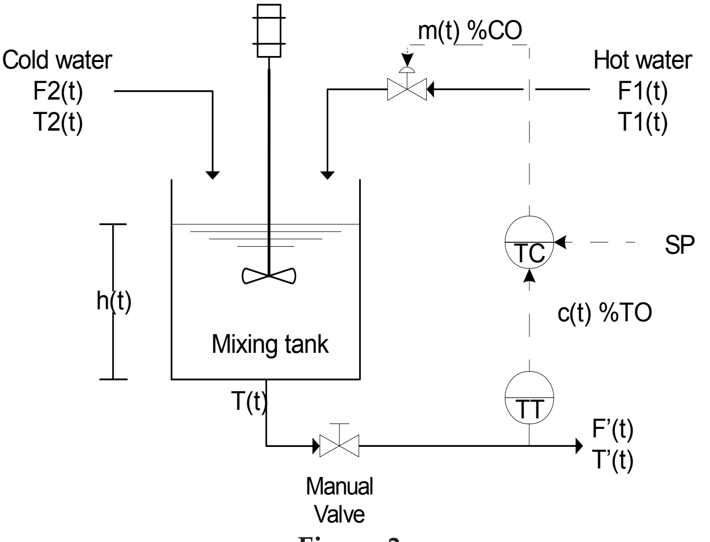

Figure 3

Schematic representation Mixing Process

A hot water stream F1(t) is manipulated to mix with a cold water stream F2(t) to obtain an output flow F'(t) at a desired temperature T'(t). The temperature transmitter is located at a distance L from the mixing tank bottom. The volume of the tank varies freely without overflowing. The mathematical model used is developed in the Appendix.

As a first step of DMCr implementation, the process was identified as a FOPDT model to determine its characteristic parameters. Fit 3 method [19] was used to perform the identification; introducing changes in the signal to the valve of +10%CO and -10%CO. The results where:

$$\left.\frac{C(s)}{M(s)}\right|_{\Delta M=+10\%TO} = \left.\frac{0.365e^{-25.35s}}{5.06+1}\right| \qquad (12)$$

$$\left.\frac{C(s)}{M(s)}\right|_{\Delta M=-10\%} = \left.\frac{0.432e^{-25.59s}}{5.60+1}\right| \qquad (13)$$

Using the model of Equation (13), and Equation (7) and the Shridhar and Cooper equations to calculate the suppression factor, the results are: λ=1.3125 and λ_{SC}=0.0653. The suppression factor determined by optimization is λ_{Opt}=1.092.

Figure 4 shows the control performance provided by each suppression factor. The Shridhar/Cooper tuning value generates a very aggressive controller behavior, a non desirable operation condition. The tuning obtained using Equation (7) produced a stable and smooth behavior, very close to that obtained with the optimum suppression factor. Table 4 shows the Integral of the Absolute Value of the Error (IAE) values for tests presented in Figure 4.

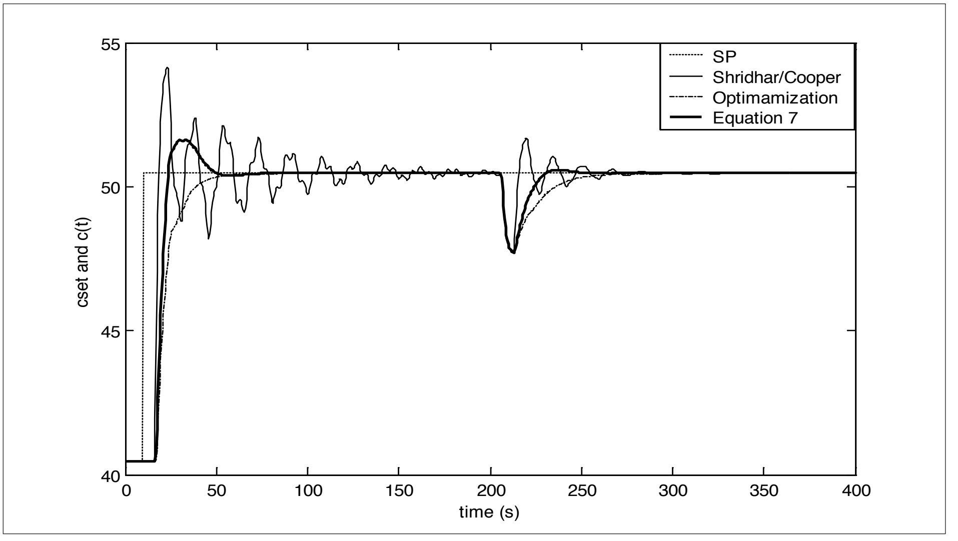

Figure 4
Performance comparison using different tuning methods for mixing process

Table 4
IAE comparison for test presented in Figure 4

Tuning Method	IAE
Shridhar/ Cooper	168.8
Equation (7)	150.6
Optimization	142.1

IAE values obtained for this test confirm that tuning parameter calculated using Equation (7) allows a smooth controller performance, very similar to that obtained using the optimal value of the suppression factor. The oscillatory behavior observed in Figure 4 corresponds to the higher IAE.

4. CONCLUSIONS

The results presented demonstrate the convenience of Equation (7) to calculate the suppression factor λ to tune DMCr. The performance of the DMCr using this tuning parameter is always smooth; tracking the set point and rejecting disturbances effectively.

References

[1] Cutler C and ramaker B. "Dynamic Matrix Control- A Computer Control Algorithm," Proceedings of the 86th National Meeting of the American Institute of Chemical Engineers, Houston, TX, US.WP5-B , 1979.

[2] Wang D. "A Simple Iterative Learning Controller for Manipulators with Flexible Joints." Automatica, Vol. 31, No. 9, September 1995. pp. 1341-1344.

[3] Steinbuch M. "Repetitive control for Systems with Uncertain Period-time". Automatica, Vol. 38, No. 12, December 2002, pp. 2103-2109.

[4] Goldsmith P. "On the Equivalence of Causal LTI Iterative Learning Control and Feedback Control." Automatica, Vol. 38, No. 4, April 2002, pp. 703-708.

[5] Arzen K. "AI in the Feedback Lop: a Survey of Alternative Approaches." Annual Reviews in Control. Vol. 20, 1996, pp. 71-82.

[6] Jones A. "Problems with Expert Systems in Real-time Control." Engineering Applications of Artificial Intelligence. Vol. 6, No. 6, December 1993, pp. 499-506.

[7] Astrom K. "Autonomous Controllers." Control Engineering Practice. Vol. 1, No. 2. April 1993. pp.227-232.

[8] Balakrishman S and Weil R. "Neurocontrol: A Literature Survey." Mathematical and Computer Modeling. Vol. 23, No. 1-2, January 1996, pp.101-117.

[9] Jelena and Nigel. "Neuro-fuzzy Control of a Mobile Robot." Neurocomputing. Vol. 28, No. 1-3, October 1999, pp. 127-143.

[10] Wong B, Lai V, Lam J. "A bibliography of Neural Networks Business Applications Research: 1994-1998." Computers and Operations Research. Vol. 27, No.11-12. September 2000, pp-1045-1076.

[11] Ge S, Li G, Lee T. "Adaptive NN control for a Class of Strict-Feedback Discrete-time Nonlinear System." Automatica. Vol. 39, No 5. May 2003, pp. 807-819.

[12] Qin SJ and Badgwell T. "An Overview of Model Predictive Control Technology," Proceedings of Fifth International Conference in Chemical Process Control- CPCV, Tahoe, California. p. 232-256 , 1996.

[13] Sanjuan M. "Control Dinámico Matricial: Fundamentos y Aplicaciones." Revista Ingeniería y Desarrollo. 1, 3-4, 1999. 85-92

[14] Sanjuan M. "Design and Implementation of Fuzzy Supervisors to Perform On-line Compensation of Nonlinearities in SISO PID Control Loops," Doctoral Dissertation, USF, Tampa. 2002.

[15] Shridhar R and Cooper D. "A Tuning Strategy for Unconstrained SISO Model Predictive Control," Ind. Eng. Chem. Res. Vol. 36. pp. 729-746. 1997.

[16] Shridhar R and Cooper DJ, "Selection of the Move Suppression Coefficients in Tuning Dynamic Matrix Control." Proc. American Control Conference, New Mexico, pp. 729-734. 1997.

[17] Dougherty D and Cooper D.J. "Tuning Guidelines of a Dynamic Matrix Controller for Integrating (Non-Self-Regulating) Processes". Ind. Eng. Chem. Res. Vol. 42. pp. 1739-1742. 2003.

[18] Iglesias E. Doctoral Dissertation in Chemical Engineering Department; University of South Florida: Tampa, 2005 (In progress).

[19] Smith C and Corripio A. Principles and Practice of Automatic process control; 2nd. ed.; John Wiley and Sons: New York, 1985.

APPENDIX

This appendix provides the mathematical model of the tank in Figure 3.

An unsteady state mass balance around the tank gives:

$$F_1(t)\rho + F_2(t)\rho - F(t)\rho = \rho \frac{dV(t)}{dt} = \rho A \frac{d(h(t))}{dt} \quad \text{(A-1)}$$

where ρ is the flow density, *A* is the tank cross section and *h(t)* is the liquid level inside the tank.

The output flow *F(t)* is modeled as a function of the liquid level and the manual valve used in the bottom of the tank:

$$F(t) = CV\sqrt{h(t)} \quad \text{(A-2)}$$

Assuming the contents of the tank are well-mixed, an energy balance around the tank gives:

$$F_1(t)\rho CpT_1(t) + F_2(t)\rho CpT_2(t) - F(t)\rho CpT(t) = \rho ACv\frac{d(h(t)T(t))}{dt} \quad \text{(A-3)}$$

where *Cp* and *Cv* are the heat capacity of the liquid at pressure constant and volume constant respectively. $T_1(t)$ is the hot water stream temperature, $T_2(t)$ is the cold water stream temperature. *T(t)* is the temperature just in the bottom of the tank.

Because the sensor/transmitter TT is located at a distance L from the tank bottom, there is a delay time between T(t) and the temperature registered by the sensor/transmitter T'(t). That delay time *to(t)* can be calculated as:

$$to(t) = \frac{LAt\rho}{F(t)} \quad \text{(A-4)}$$

where *At* is the pipe cross sectional area, and *L* is the distance between the tank bottom and the sensor/transmitter position.

The temperature registered by the sensor/transmitter can be related to the output temperature as:

$$T'(t) = T(t\text{–}to(t)) \quad \text{(A-5)}$$

The sensor/transmitter is modeled as a first order differential equation:

$$\tau_T \frac{dc(t)}{dt} = c(t) = K_T(T'(t) - T_{\min}) \quad \text{(A-6)}$$

where τ_T and K_T are the sensor/transmitter time constant and gain respectively. T_{min} is the minimum reading of the sensor/transmitter. $c(t)$ is the output signal.

The control valve used to manipulate stream $F_1(t)$ is also modeled as a first order differential equation:

$$\tau_v \frac{dF_1(t)}{dt} = F_1(t) = K_v m(t) \quad \text{(A-7)}$$

where τ_V and K_v are the time constant and gain of the valve respectively.

Table 3 shows all steady state values of all the variables.

Table 3
Steady state values for mixing process

Parameter	Steady State Values	Units
F_1	0.8	m^3/s
F_2	1.1	m^3/s
F	1.9	m^3/s
T_1	80	^oC
T_2	15	^oC
T	42.36	^oC
ρ	1000	kg/m^3
V	10	m^3
Cv	1	$kcal/^oC\text{-}kg$
Cp	1	$kcal/^oC\text{-}kg$
CV	0.6	$m^3/m^{0.5}$
L	3	m
τ_V	0.5	s
K_v	0.016	$(m^3/s)/(\%CO)$
K_T	1.25	$\%TO/^oC$
τ_T	0.5	s